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Contents

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Executive Summary

The non-linearity due to Kerr effect in optical fiber channel is considered as the main limiting factor for achieving high data rates. The presence of non-linear distortion in the optical channel is the fundamental difference between fiber optical communications and other wire-line technologies, e.g., transmission over coaxial cables. Therefore, it is of importance to understand the influence of this effect and design the transmission system based on that.

In WP1, we are studying the design of efficient fiber-optical systems for uncoded and coded transmission scenarios. The aim is to increase data rates in fiber-optics. In order to achieve this goal, we start by studying the state of the art to understand the advantage and disadvantages of previous works, and then based on this knowledge we will try to design some novel transmission schemes, including new modulation formats and detection techniques to improve the data rates of the system.

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1 ESR 1

1.1 Optical Fiber Channels

A communication channel refers to the physical transmission medium in the communication system, which the engineer is unable or unwilling to change [1]. As a channel performance measure, the channel capacity is defined as the maximum data rate (usually in bits/s, or in bits/s/Hz if normalized by bandwidth) at which one can reliably transmit information over the channel [2]. Any other data rates, e.g., achieved by experiments or simulations under certain simplifications, should be and are referred as the achievable information rate (AIR) in this report.

In the middle and long haul optical fiber communication, the optical fiber channel is commonly considered to have standard single-mode fibers (SSMFs) and in-line amplifiers (either lumped erbium-doped fiber amplifiers (EDFA) or distributed Raman amplifiers) [3] [4]. Signals propagating in the optical fiber channel experience deterministic and stochastic distortions that arise respectively from the dispersive and nonlinear nature of SSMFs and the amplified spontaneous emission (ASE) noise of in-line amplifiers. Interactions between these effects only make the optical fiber channel more incomprehensible. Distortion compensation schemes, such as electronic dispersion compensation (EDC), digital back-propagation (DBP), optical phase conjugation (OPC) or nonlinear Fourier transform (NFT), have been under developing for decades. In spite of the enduring effort to overcome the signal distortions mentioned above, the AIR of the fiber channel demonstrates a saturating tendency beyond certain input powers [1], as opposed to the monotonic trend of the capacity of the AWGN channel against the input power.

Depending on whether the fiber channel is a part of optical fiber routed network or a point-to-point communication subsystem, we have two slightly different channels:

- **The optical fiber interference channel**. In a non-cooperative network based on wavelength-division multiplexing (WDM), each user occupies certain linear spectrum (a range of wavelengths), with transmitter under the average and peak power constraints. The users are subject to inter-user interference due to nonlinear effects. The data rate of the user of interest (UOI) is commonly considered to have a maximum. The unavailability of the interferences from other channels is due to the random adding/dropping of users in the network as shown in Fig.1 .

- **The point-to-point fiber channel**. This channel assumes that the whole spectrum of all users is available to either one or multiple receivers at the end of the link, and the cooperation between transceiver pairs is possible. Upper and lower bounds exist in [3] [4] [1], which are loose at high powers.

The goal for ESR 1 is to investigate possible native nonlinear transmission schemes and study the capacity limits of the optical fiber channels.

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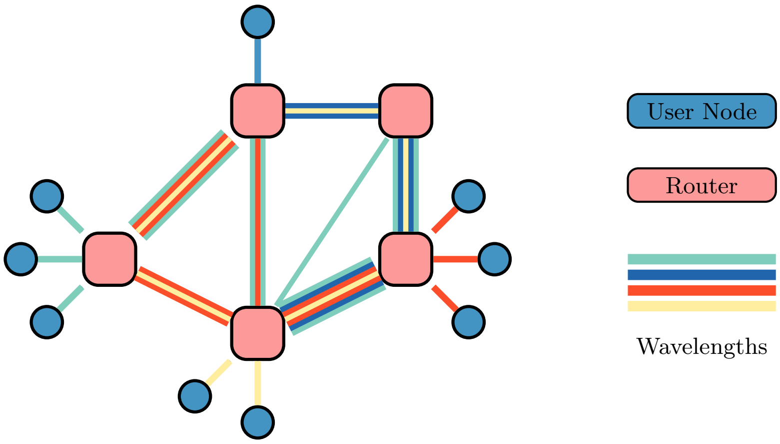

Fig. 1: WDM-based optical routed network.

1.2 Existing Native Nonlinear Transmission Scheme

It is well known that nonlinear interactions pose a limitation to the AIRs in WDM optical networks [5]. Nonlinear frequency-division multiplexing (NFDM), a signal multiplexing scheme based on NFT, is a promising approach to overcome the "capacity crunch" problem in WDM optical systems [6–8].

The nonlinear Fourier spectrum consists of a continuous component and a discrete component. Information transmission using the continuous spectrum is studied in [8–15], while discrete spectrum modulation is studied in [8,16–18]. Recent experimental demonstrations of data transmission based on NFT include a joint discrete and continuous spectrum modulation [12]. A record data rate of 32 Gb/s was demonstrated using the continuous spectrum of NFT and the 32QAM modulation format, a peak gain of 1.3 dB in SNR was achieved over a comparable OFDM system [19]; for comparison, see also [20–26]. Dual polarization transmission technique based on NFT emerged recently, utilizing the continuous and discrete spectrum in [27] [28], respectively.

Previous works have mostly applied NFT to point-to-point links, often as a nonlinear compensation method. However, the main potential of the NFDM scheme is realized in network environments. Here, users' signals are multiplexed in the nonlinear Fourier spectrum in disjoint intervals and, in the absence of noise, propagate independently in the network. Crucially, the signals UOI will not be distorted by co-propagating signals. As a result, the deterministic inter-symbol and inter-user interference are simultaneously zero for all users of a network.

Research in NFDM initially demonstrated proof-of-concepts, showing how this technique works in point-to-point channels. However, the AIRs of the NFDM signals were at best the same as the AIRs of the WDM signals in these initial demonstrations [8]. Advances in numerical methods made it possible to multiplex signals in the nonlinear Fourier domain and explore the NFT at high powers. The AIRs of NFDM and WDM were compared for the first time recently in [29,30]. It was shown that NFDM achieves data rates higher than

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WDM rates, subject to the same power and bandwidth constraints. Furthermore, while the AIR of WDM diminishes when the input power is increased past an optimal value $\mathcal{P}^*$, the AIR of NFDM increases for the input powers greater than $\mathcal{P}^*$ in simulation studies.

1.3 Open Problems

Similar to other research, NFDM raises more questions than it solved. For NFDM to be viable in commercial systems, the following problems are of great interest:

- *Improving the AIR in bits/s of NFDM*
  The most useful figure of merit for a communication system is AIR in the unit of bits/s or bits/s/Hz, but existing work rarely reported the AIRs of NFDM in bits/s or bits/s/Hz. And when they do, it usually turned out to be very low compared with WDM [16] [29].

- *NFT with periodic boundary conditions*
  For both dispersive and solitonic waves, a guard interval in time-domain is needed between two signal blocks due to the vanishing boundary condition of NFT, which leads to a lower AIR in bits/s than the AIR of conventional systems with Nyquist pulses or root-raised cosine pulses. Using NFT with periodic boundary conditions could potentially solve the problem. Existing works on the periodic nonlinear Fourier transform (PNFT) [31] [32] demonstrated proof-of-concepts using the analytical solutions of certain solitonic waves. An algorithm for the inverse PNFT that enables the modulation of arbitrary intervals of the nonlinear spectrum does not exist yet.

- *Computing NFT using optical signal processing*
  In an NFDM network, the add-drop multiplexer (ADM) should have the ability to add or drop users, which occupy disjoint nonlinear spectra, without optical-electrical and electrical-optical conversion, in a similar way how the traditional optical ADMs deployed in the WDM system work.

- *Computational complexity issue of NFT and INFT algorithms*
  The computational complexity of INFT regarding the number of samples N in time and eigenvalues K is $O[KN + N\log^2 N]$ [33] (in the defocusing regime $K = 0$). Lima et al. investigated the required number of samples in the time domain per processing frame for several values of the average input power in the defocusing regime [34]. The processing frame is defined as the pulse duration at the Rx, which is typically longer than pulse duration at the Tx. It is argued that a prohibitively large number of signal samples is needed at high powers for NFDM to be viable in practice. The complexity issue can be addressed either by a novel analog optical component or a simpler algorithm.

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1.4 Project Tasks

With all the above-mentioned problems in mind, we found it important to further develop PNFT. The reasons are: 1) despite the fact that NFDM substantially reduces the interferences between co-propagating users, the AIR in bits/s of each user is limited by the vanishing boundary condition of NFT. The development of PNFT becomes vital for the successful application of the inverse scattering theory in optical communication systems, 2) complexity issues should only be addressed after a higher spectral efficiency achieved by a (P)NFT-based system has been demonstrated, 3) building the nonlinear ADM as stated in the third open problem is also essential, it is, however, beyond the scope of the communication engineering.

In addition to developing the existing nonlinear transmission scheme, other alternatives should also be sought for. Digital signal processing with machine learning techniques, especially neural networks, appears to have a great potential [35] [36]. It can be easily shown that a simple two-layer neural network can be trained to compensate chromatic dispersion. K. Hornik proved in [37] that standard multilayer feed-forward networks with as few as a single hidden layer and arbitrary bounded and non-constant activation function are universal approximators with respect to $L^p(\mu)$ performance criteria, for arbitrary finite input environment measures μ, provided only that sufficiently many hidden units are available. It is, therefore, possible that a neural network with a large number of layers can be trained to compensate both dispersion and nonlinearity with less computational complexity.

The ESR 1 will also approach the problem of the ultimate capacity of the fiber optical channel. Due to the lack of a comprehensible channel model and the existence of in-line noise, a novel method needs to be found.

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2 ESR 3

2.1 State of the Art

A channel refers to a physical transmission medium, and it is used to convey information signal and it is described with a mathematical model. However, in many cases, the mathematical model does not accurately represent the reality. In the context of fiber-optical communications, one of the fairly accurate channel models for single-mode fibers (SMFs) is nonlinear Schrödinger equation (NLSE), which is derived from Maxwell's equations under some assumptions and approximations. The NLSE is a deterministic channel model which is derived for a single span link. Moreover, it is a partial differential equation which represents the input-output relationships for optical baseband signals. The presence of nonlinear distortion in the channel is the fundamental difference between optical communications and radio frequency (RF) communications. As a result, it is important to understand the behaviour of this effect and design the transmission system based on that. There many works in the literature which examine this nonlinear phenomena. Among them, we will pinpoint the three most relevant studies to our project.

- In [38], the authors investigates the design of amplitude phase-shift keying (APSK) constellations for a coherent fiber-optical communication system where nonlinear phase noise (NLPN) is the main system impairment. The design of signal constellation considering the fiber nonlinearity is studied in this paper. The optimum design of a signal constellation is placing M points in the complex plane such that the symbol error probability (SEP) is minimized under an average or peak power constraint. In this paper, the authors focused on APSK signal constellation. Moreover, they considered a simplified dispersionless channel model which follows from the NLSE by neglecting the dispersive term. They used a discrete channel which has been considered previously by several authors. Since all of the derivations neglect dispersion, the resulting PDF should serve as a useful approximation for dispersion-managed (DM) optical links.

  The detection technique which is used in [38], includes two stages, and it is a practical alternative to the ML detector. In particular, the TS detector employs one-dimensional decisions: First in the amplitude direction (first detection stage), followed by a phase rotation, and then in the phase direction (second detection stage). These two stages together are called *postcompensation*. At the first stage, based on the absolute value of the observation, radius detection is performed. Due to the fact that each ring has a different number of symbols, the authors used maximum a posteriori (MAP) instead of ML radius detection and make use of a priori probability for selecting a certain ring at the transmitter. At the second stage, a phase detector chooses the constellation point with detected radius.

  Beside the TS detection, the parameters of APSK modulation are optimized under TS detection. Due to the fact that the symbol error probability (SEP) under TS detection

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is independent of the phase vector, it does not appear in the optimization problem. For $M = 16$ constellation points, significant performance improvements in terms of SEP can be achieved by choosing an optimized APSK constellation compared to a typical 16-QAM constellation.

- In [39], signal design and detection for a zero dispersion channel model are investigated. The authors consider the distributed noise scenario for long haul links. They study signal design and implementation of ML detection for phase-modulated systems. For PSK and DPSK systems, they present an approximate phase postcompensation, which has the form $c_2r^2 + c_1r + c_0$, where r is the radius of observed signal. After this post compensation, the ML decision boundaries will be approximately similar to those of AWGN Channel.

  In [39, Sec 3], the authors investigate the QAM signal design and ML detection strategies. They seperately study systems with low and high nonlinearities and propose various phase rotation techniques at the transmitter and receiver to approximate the ML detection using straight-line decision boundaries.

  In the low nonlinearity regime, a 16-QAM transmitter setup is considered. With this constellation setup, it is not possible to perform optimal phase postcompensation, as the transmitted power is not constant for all the signals. However, for 16-QAM systems with low degree of nonlinearity, the decision boundaries are close to straight lines, and as a result, the authors design some precompensation and postcompensation techniques for this scenario such that the ML detection boundaries are well approximated by straight lines.

  Finally, In [39, Sec 4], a new design for signal constellation with a given number of signal points, and an average transmit power constraint is proposed. The authors consider the four point signal constellation optimization with some symmetry constraints such that the problem is more tractable and the solution is more attractive for practical implementation purposes.

- The autocorrelation function of the output signal given input signal for a dispersion-free fiber channel is derived in [40]. The per-sample statistics do not consider the spectral broadening. In fact, the propagation signal bandwidth, grows with the launch power. Moreover, a per-sample receiver has infinite bandwidth while practical receivers are bandlimited. In [40], the author studied the two-sample statistics for the waveform channel with some practical constraints on optical amplifiers, transmitter, and receiver.

  In [40, Sec 5], the autocorrelation function in closed form for the dispersion-free optical fiber with distributed optical amplification is derived. Moreover, the autocorrelation is computed for the rectangular pulse shaping. This mathematical closed form, is the main result of the paper, which gives a bound on the output power of bandlimited and time-resolution limited receivers.

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2.2 Open Problems

The main goal of WP1 is to find some novel transmission schemes for fiber optic communications in nonlinear regime. It is expected that the outcome will be the experimental proof of modulation formats and detection schemes for single span and multi span links (i.e., for metro and long-haul links). To approach this goal, we identified the following open problems.

- One of the interesting problems in fiber optics which is not effectively addressed yet, is finding an alternative for matched filter. It is known that the matched filter is optimum for linear channels. However, in nonlinear channels, it is not necessarily optimum. As a result, one can do research to address the mentioned problem and find an alternative process to outperform matched filtering.
- Long-haul telecommunications is an important research area which requires high-capacity trunk lines. One of the issues regarding long-haul telecommunications, is the attenuation of the signal in optical fiber which is in practice addressed by repeaters (e.g., optical amplifiers). In practice, the lengths of spans in a multi-span communications are similar, however, the optimum span lengths and amplifier gains depend on many factors such as the modulation format (i.e., the transmitted waveforms for each symbol), the number of spans, the channel model, and the detector which is used in the transmission system.
- Another interesting problem regarding WP1, is designing new modulation waveforms for fiber optics communications, which leads to lower SER. One might want to find the optimal signal constellation set that minimizes the SER. However, it is known that for non-gaussian noise statistics, no analytical results for the optimal signal set exist [41], and numerical methods that are developed in solving such optimization problems are rather complicated [42].

2.3 Project Tasks

The analytical channel models for the nonlinear optical channel available in the literature may not be suitable for the design of transmission system. One way to deal with these kind of problems is to start with simplified models. As a result, at the beginning of this project, we will study the more simplified and mathematically tractable channel models, modulation formats and detectors, and then we will try to consider more realistic channel models and system setups to apply the results to them. During this process, normally, some new open problems will be identified.

To address one of the open problems regarding to detection of transmitted symbols, the following roadmap will be considered.

- *One sample per symbol detection:*

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In this scheme, each symbol is detected by just one sample. The per-sample model is attractive since for zero-dispersion fiber channel, there is closed form expressions for the statistics. The aim of studying this scheme is to use the results for detection of more realistic schemes with multiple samples per symbol.

- *Multiple samples per symbol detection*

  It is known that in linear systems, the matched filter is optimum. However, for nonlinear channels, it is not optimal. Therefore, one should investigate alternative processes for these kind of channels.

- *Multiple symbols detection (Intersymbol interference)*

  For channels with dispersion or for bandlimited pulse shapes, the effect of intersymbol interference in symbol detection should be taken into account.

- *Predistortion to mitigate nonlinear distortion*

  The standard linear modulator will be expanded with nonlinear correction terms, so that the received signal after propagation on the nonlinear fiber will be more similar to a linear combination of transmitted pulses, and thus suitable for a standard matched-filter receiver. Such a transmitter-side signal expansion will be first optimized numerically and then theoretically analyzed and refined into a generalized signal-space description.

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