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Executive Summary

0.1 Summary of ESR 2

This document contains a summary on the conducted research of ESR2 under WP2 under the COIN project. Potential future directions of the work with regard to optimum detection and machine learning techniques applied for optical fibre communication are discussed.

The increasing information rate demands on optical fibre links require Nyquist-spaced wavelength division multiplexing (WDM) combined with advanced dual-polarisation modulation formats. However, the main challenge for increasing the transmission rates of such systems is imposed by the Kerr nonlinearity in the medium, which causes power-dependent distortions of the signal. The effects on system performance are more severe in cases where wider transmission bandwidth, closer channel spacing and high order modulation formats are utilised. Therefore, optimised transmission schemes, tailored to the fibre nonlinearity, play an integral part in achieving high data rate optical fibre communication.

The report starts with a description of two possible techniques for increasing the information rates in transmission systems distorted by nonlinearity,namelydigital backpropagation (DBP) and maximum likelihood sequence detection (MLSD). Nonlinearity leads to spectral broadening in the output signal which is also descried and quantified. The report gives a brief overview of the literature and research progress to date as well as the directions for future work towards optimum detection and also the use of machine learning techniques for fibre-optic communication.

0.2 Summary of ESR 4

This document contains a survey on existing literature and previous work in the field of the nonlinear Fourier transform (NFT). Potential open problems and research areas with regard to coding and coded modulation are identified and methods to solve them are suggested.

It focuses on the NFT, a concept utilizing the nonlinearities of the fiber optic channel for the design of the communication system. Pulse propagation in optical fibers is governed by a nonlinear differential equation - the nonlinear Schrödinger equation (NLSE). The corresponding nonlinear effects on a transmitted pulse of light severely impair communication over this channel. The NFT transforms a signal from the time domain into a nonlinear spectral domain where the modes propagate linearly, a characteristic that is highly desirable for a communication system.

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1 ESR 2

1.1 Techniques for increasing information rates in optical transmission systems

The achievable information rate is a natural figure of merit in communication systems which demonstrates the obtainable data rate for a given transmission scheme. Significant increases in information rates in optical communication systems can be achieved by expanding the communication bandwidth while in the same time increasing the spectral efficiency of the transmission. The advent of new wideband transceivers and optical amplification schemes, as well as new types of optical fibres with ultra-low loss performance would lay the foundations to the extension to new transmission windows such as L (1565-1625 nm) and S(1460-1530 nm) bands in addition to the conventional C-band (1530-1565 nm) communication. On the other hand, greater spectral efficiency is realised by the utilisation of higher-order dual-polarisation in-phase/quadrature modulation formats, which, however, need higher signal-to-noise ratio at the receiver to achieve reliable communication. This requires an increase in the launch power into the fibre, which, as a result during signal propagation, gives rise to the power-dependent nonlinear distortions and thus significant performance degradation. Therefore, advanced detection and signal processing techniques at the receiver side need to be implemented to tackle the limitations imposed by fibre nonlinearity.

System performance in presence of nonlinear impairments can be improved from the perspective of optimally designed digital signal processing techniques that undo the channel distortions and thus equalise the received signal. Another approach is to implement an optimised detection strategy that outputs the most probable estimate of the actual transmitted data given the observations on the received signal. In the following, the most currently widespread equalisation technique digital backpropagation (DBP), and optimum detection scheme maximum likelihood sequence detection (MLSD), are described.

The Gaussian noise (GN) channel model is a simple and widely-accepted model of nonlinear fibre propagation [1]. Distortions arising from fibre nonlinearity are modelled as an additive source of Gaussian noise.This is a valid assumption for operation at optimum launch powers in dispersion-unmanaged links. As a result this noise the added, in power, to the amplified spontaneous emission (ASE) noise from the optical amplifiers. The expression for the effective SNR then becomes [2, 3]:

$$SNR = \frac{P}{\sigma^2_{ASE} + \sigma^2_{S-S} + \sigma^2_{S-N} + \sigma^2_{S-TR} + \sigma^2_{TR}}. \quad (1)$$

where σ^2_{ASE} is the noise from amplified spontaneous emission, σ^2_{S-S} is the contribution of signal-signal nonlinear interactions, σ^2_{S-N} accounts for the signal-ASE noise beating, σ^2_{S-TR} is the nonlinear interaction of transmitter noise and the signal and σ^2_{TR} is the transceiver noise term. Due to the trade-off between ASE noise and nonlinear interactions there is a nontrivial value of the optimum span length that minimises the noise contributions in the denominator of Eq. (1). For instance, the span length problem has been studied in terms of energy optimisation [4, 5], nonlinear phase shift in hybrid Raman-EDFA-amplified on-off keying systems [6], and Q-factor in quadrature phase-shift keying (QPSK) ultra-long haul submarine systems [7]. However, the impact of modulation format and nonlinear compensation on the

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optimal span length have not been reported in the literature.

1.1.1 Digital backpropagation (DBP)

Digital backpropagation is an equalisation technique which tries to undo the signal distortions by backpropagating the whole (full-field DBP) or part (partial bandwidth DBP) of the received spectrum [8, 9]. It benefits from the fact that the propagation of optical pulses in fibre medium can be relatively accurately described by the nonlinear Schroedinger equation which is a second-order nonlinear partial differential equation, whose solution can be obtained through numerical method such as the split-step Fourier method. Therefore, DBP can be used to compensate, or at least, mitigate the deterministic nonlinear effects described by the propagation equation. In particular, it compensates for the signal-signal nonlinear interactions and when full-field DBP is applied the σ^2_{S-S} noise contribution term in Eq (1) vanishes. Applying DBP increases the SNR performance of the system and shifts the optimum launch power towards higher values. For example, in [3] it has been demonstrated experimentally that full-field DBP gives 2 dB in SNR gain over electronic dispersion compensation (EDC) scheme for 4-channel 30 Gbaud WDM system with 32 GHz channel spacing modulated with DP-64QAM at total distance 5000 km (100 km/span).

However, the nonlinear interactions between signal and noise have a stochastic nature and cannot be compensated by DBP and, therefore, limit the performance of the communication system. Such interactions arise from the beating of the transmitted signal with the amplified spontaneous emission (ASE) noise from the optical amplifiers. Another source of transmission noise which interacts nonlinearly with the signal during propagation is the transceiver noise which is present in practical systems [3].

1.1.2 Maximum likelihood sequence detection (MLSD)

Classical detection strategies at the receiver implement matched filtering and symbol-by-symbol decisions. However, the dispersion effects in the transmission fibre introduce memory in the channel and therefore multiple received symbols have an impact on the symbol of interest. It has been proven that maximum likelihood sequence detection (MLSD) is an optimum way of designing a receiver which accounts for the memory in the channel [10]. MLSD is usually implemented to minimise the Euclidean distance between a noisy received symbol sequence and pre-trained noiseless reference. It is implemented using the Viterbi algorithm. For fibre transmission, even after dispersion compensation the interactions between fibre nonlinearity and dispersion impose a residual memory, i.e. the received symbol depends on the state of its neighbours, and, hence, detection scheme which takes into account this memory needs to be considered.

An integral part of achieving optimum detection with MLSD is to collect sufficient statistics about the waveform for the implementation of the Viterbi algorithm [11]. It has been shown that for single span single channel 32 Gbaud QPSK SSMF transmission the bit-error-rate of the system can be monotonically decreasing with power at all examined distances up to 350 km. In general, because of the nonlinear propagation, the received bandwidth of the signal is expanded compared to the transmitted one, an effect known as spectral broadening. Therefore applying a filter matched to the transmitted signal is not optimal for the nonlinear fibre channel. This

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has been highlighted by the importance of collecting sufficient statistics, i.e. ensuring that the received waveform samples enable optimum operation of the algorithm, as it is shown that combining matched filter with the MLSD can be significantly outperformed by first applying wide bandwidth rectangular low pass filter to capture the full spectrum and then performing the algorithm. Such a receiver structure can be claimed to be optimal as long as the detection memory matches the memory of the channel.

1.1.3 Spectral broadening

The spectral broadening of the transmitted signal due to nonlinear interactions has an important impact on both the DBP and MLSD techniques. It arises from the intra- and inter-channel signal nonlinear interactions [12, 13]. Apart from causing performance degradation, the presence of spectral broadening causes the uncertainty in the design of the appropriate matched receiver filters for optimum detection and the calculation of achievable spectral efficiency in optical communications [10]. The spectral broadening effects have been investigated in optical communication systems without using any nonlinearity compensation. The power spectral density of XPM-induced spectral broadening has been analysed in [14], and the impact of spectral broadening on the transmission performance of WDM systems has been evaluated in [15]. An in-line compensation of the spectral broadening in standard single mode fibre (SSMF) using dispersion compensating fibre was also demonstrated in the dispersion-managed systems [16]. However, the spectral broadening effect in nonlinearity-compensated transmission systems and its impact on the nonlinearity compensation have never been reported. Interestingly, the impact of signal spectral broadening on the performance of MC-DBP has never been quantified and must be considered to explore the effectiveness of nonlinearity compensation, channel tailored receiver design, and the estimation of achievable spectral efficiency.

1.2 Research progress to date

During the period October 2016-July 2017 research focused on maximising the information rates of nonlinearity degraded optical communication systems is presented. The key activities and results are described here and the resultant publications are listed at the end of the reference list.

Increasing the transmission bandwidth is an integral part of achieving higher transmission rates. However, impairments due to nonlinear interactions are more severe impact for systems with larger bandwidths, closer channel spacing and higher order modulation formats. We examined the impact of modulation format on nonlinearity compensation, specifically digital backpropagation, in fully-loaded C-band communication systems from the perspective of the achievable information rate [17]. The investigation consists of simulating 151-channel 32 Gbaud Nyquist-spaced transmission over multiple spans of SSMF with DP-QPSK/DP-16QAM/DP-64QAM/DP-256QAM modulation formats applied at distances ranging from 400 to 10 000 km. The bandwidth of nonlinearity compensation ranges from 32 GHz (single-channel) to 4.8 THz (full-field). Transceiver limitations were considered in the simulation to obtain performances of practical systems with nonlinearity compensation. It was found that modulation format and transmission distance determine the efficacy of nonlinearity compensation in enhancing the information rates. In particular, for higher order modulation format

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the distance at which nonlinearity compensation starts to increase the information rates is shorter. It was found that DP-QPSK does not require nonlinearity compensation for distances up to 10 000 km. For DP-16QAM this distance is 2 000 km, while DP-64QAM and DP-256QAM benefit from nonlinearity compensation at distances longer than 400 km. This work gives an insight on the application of nonlinearity compensation in wide bandwidth optical communication systems with transceiver limitations.

In [18] we have further extended the investigation of multi-channel DBP for spectrally efficient systems taking into account the optimisation of the algorithm in terms of the minimum required number of steps per span (MRNSPS) parameter. Again DP-QPSK, DP-16QAM, DP-64QAM and DP-256QAM modulation format are considered in 9-channel 32 GBaud Nyquist-spaced transmission over 2000 km of SSMF and the nonlinearity compensation bandwidth ranges from single channel (32 GHz) to 9 channel (full field, 288 GHz). Our study shows that depending on whether the SNR or the information rate of the system is maximised, the MRNSPS can significantly vary. The comparison between SNR-optimised and information-rate-optimised MRNSPS shows that the conventional SNR optimisation often overestimates the system requirements. For example, full-field DBP maximises the AIR of the DP-64QAM system when it is performed at 250 steps/span, while the SNR is maximised at 500 steps/span. Therefore the complexity of applying multi-channel DBP may be considerably reduced if the AIR instead of the SNR is maximised for a given modulation format. Furthermore, our investigation shows that for a given AIR there exists a potential trade-off between the modulation format and backpropagated bandwidth and hence a compromise may be achieved according to practical complexity limitations. For instance, the AIR achieved at optimum power by DP-256QAM and 7-channel DBP are higher than the rate for full-field (9-channel) DBP and DP-64QAM.

currently single-channel DBP remains more practically feasible nonlinearity compensation scheme. It was already highlighted that at optimum span lengths the transmission noise is minimised and as a consequence the highest information rates are achieved. In [19] we investigate the span length choice issue raised by single-channel DBP in terms of information rates for different high order considering practical transceiver limitations in EDFA-amplified systems. Reductions in the number of amplifiers, at fixed information rates, were investigated from the perspective of applying single-channel digital backpropagation instead of compensating only linear channel impairments. A representative 11-channel 32 GBaud Nyquist-spaced system was simulated with DP-QPSK, DP-16QAM, DP-64QAM and DP-256QAM modulation formats applied. The span length of SSMF was varied from 20 km to 120 km at three different total link distances: 2400, 4800, and 7200 km. The maximum achievable information rate at optimal span lengths has been studied for the cases of linear, electronic dispersion compensation (EDC), and nonlinear, single-channel DBP, compensation. Practical schemes that assume transceiver noise limitations were considered to obtain realistic nonlinear compensation gains. It was found that at shorter spans these gains are more significant and decrease with span length. It was shown that inter-amplifier spacing can be doubled at the same information rates as in span-length-optimised systems with linear compensation. Consequently, implementing single-channel DBP can effectively reduce the total number of amplifiers in the system by 50% without sacrificing information rates. Such an approach can lead to significant cost savings in commercial submarine and terrestrial optical transmission systems.

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The impact of spectral broadening, as an effect of fibre nonlinearity, is an important consideration for both DBP and MLSD. In [20] we investigate the performance of multi-channel DBP when signal spectral broadening is considered. Numerical simulations have been carried out in 32-Gbaud single-channel/multi-channel Nyquist-spaced WDM standard single-mode fibre transmission system. A variety of modulation formats including dual-polarization QPSK (DP-QPSK), DP-16QAM, and DP-64QAM have been considered to study the impact of modulation format on spectral broadening. It is found that signal spectral broadening must be taken into account to achieve optimal nonlinearity mitigation, for which the full detection of broadened signal spectrum is required. This applies to both single-channel and multi-channel optical communication systems, and is independent of signal modulation formats. For multi-channel systems, the degradation of MC-DBP performance, when the spectral broadening effect is not considered in the nonlinearity compensation, is more significant for outer channels. For single-channel system, the penalty in terms of the best achievable signal-to-noise ratio (SNR) is 2.8 dB for 800 km transmission, and 1.8 dB for 2000 km transmission. For the outer channels in the 5-channel system, the SNR degradation is 1.9 dB for 800 km transmission and 1.0 dB for 2000 km transmission. In addition, the signal spectral broadening effect was evaluated numerically for transmitted signal bandwidths varying from 32 GHz to 800 GHz. The investigation is used to quantify the minimum bandwidths of optical receivers and signal processing devices to ensure the optimal compensation of deterministic nonlinear distortions. It is shown that optical receiver and signal processing devices require additional bandwidth (32 GHz - 48 GHz in our scheme) beyond the transmitted signal bandwidth to guarantee complete compensation of deterministic nonlinear distortions in Nyquist-spaced optical transmission. A study on the impact of transmission distance on the signal spectral broadening has also been carried out. To make a fair comparison between different numbers of spans, the ASE noise in the EDFA within each fibre span was not included. It is shown that the spectral broadening effect increases with the transmission distance. Interestingly, the rate of increase becomes lower for longer transmission distances, which is due to the accumulated dispersion in the transmission link gradually weakening the accumulation of the spectral broadening effect over multiple transmission spans.

A receiver built on the MLSD principle is optimum to deal with channels that exhibit memory such as the optical fibre channel. From the perspective of finding the transmission rates limits of fibre-optic communication, the single span channel is a relevant issue since a complete regeneration at the end of each span in multi-span systems clearly represents an upper bound on the achievable information rates. In [21] we investigate the structure of an optimum receiver for single-span fibre system where the ASE noise addition typically happens at the end of the span. We demonstrate that because of the absence of signal-noise nonlinear interactions, provided the optimum detection is implemented, it is possible to achieve monotonically decreasing BER performance with increasing launch power even without compensating the channel nonlinearity. The optimum receiver described in the manuscript consists of a wide bandwidth (32 GHz) rectangular low-pass filter (RLPF), which captures the whole broadened spectrum of the signal, followed by sampling at the Nyquist rate and implementation of the MLSD algorithm. In such a way sufficient statistics are provided to the estimation algorithm. The sub-optimality of other receiver structures such as the matched filter symbol-by-symbol (MF symbol-by-symbol) detection and matched filter followed by MLSD (MF MLSD) is also highlighted. In

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particular our research showed that for 32 Gbaud single channel QPSK transmission over 350 km of SSMF the optimum receiver structure implemented with RLPF 7-symbol MLSD achieves monotonically decreasing BER for all examined launch powers up to 28 dBm. On the other hand, the BER of the MF 7-symbol MLSD design increases when the launch power is higher than 21 dBm, thus emphasising on the importance of collecting sufficient statistics for the estimation and sub-optimality of applying a filter matched to the transmitted signal. For the examined scenario utilisation of the optimum receiver design allows 125% increase in achievable information rates (from 25 GBit/s to 60 Gbit/s) at 350 km distance in comparison with the conventional MF symbol-by-symbol detection. Such significant gains give a strong reason to consider optimum receivers for increasing the information rates in fibre-optic communication systems.

1.3 Outlook

Optimum detection tailored to the fibre nonlinearity is an interesting and fundamentally important research topic that needs further development in directions such as extending optimum receiver techniques to multi-channel scenarios, which would significantly increase the complexity of the estimation algorithm as the memory of the channel is increased. Therefore, reduction in complexity through decrease in the number of states number of states in the Viterbi Trellis without sacrificing performance is an essential part of the future work. Applying higher- order dual-polarisation modulation formats would certainly benefit from such a complexity reduction. For the extension of the MLSD techniques to multi-span systems the detection strategy needs to be modified significantly as the signal-noise interactions need to be taken into consideration. Therefore, knowledge of their statistical properties would be required.

Applying machine learning techniques for communication over the optical fibre is a future research direction under WP2 ESR2. In particular, simple deep learning techniques such as feedforward neural networks can be an interesting alternative to conventional equalisation schemes. The performance of such strategies is currently under investigation.

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2 ESR 4

2.1 The Nonlinear Fourier Transform

The NLSE is an integrable differential equation and belongs to the class of nonlinear integrable systems. For this class of systems, the mathematical tool of the inverse scattering transform can be used to find solutions [22]. Pulses on the fiber corresponding to these solutions propagate unaffected by the nonlinearities, a fact that is highly desired regarding a communication system. Signals with this characteristic are referred to as solitons and have been studied extensively in literature [23, 24]. Hasegawa and Tappert proposed to use solitons to embed information in[25]. They extend it to the use of higher order solitons and perform investigations of the non-ideal case, i.e., lossy transmission with periodic amplification [26]. The design of a wavelength division multiplexing (WDM) system using solitons is demonstrated in [27]. The potential of solitons is shown in [28] where data transmission over one million kilometers is performed experimentally. Over time, an extensive amount of research has been conducted to characterize solitons and study their behavior under the influence of noise and components of an fiber optic communication system, e.g., filters, multiplexer or switches. An overview over dispersion managed solitons is given in [29]. It introduces the mathematical background and shows concepts for practical implementations. In [30] an estimate of the lower bound of the capacity of a soliton transmission system is given. In [31, 32], the statistic of a soliton under the presence of amplifier-induced spontaneous emission (ASE) noise are derived and in [33], the influence of additive noise and in-line elements such as filters is investigated.

In the work of Mansoor and Kschischang [34], the concept of the inverse scattering transform is put into an information theoretic context. Instead of the term inverse scattering transform, the name NFT is used to highlight the similarity to the Fourier transformation. The NFT transforms a signal into its solitonic components and its scattering data. The solitonic part is referred to as nonlinear discrete spectrum and the scattering components as nonlinear continuous spectrum. In [34], the mathematical background, numerical methods and potential applications to a communication system are given. The inverse, i.e., transforming the nonlinear spectrum (discrete and continuous) to the time domain, is referred to as inverse nonlinear Fourier transform (INFT) and also described in [34]. A number of different numerical methods for calculating the NFT have been proposed. We give a short overview but refer to the according references since algorithms for the NFT are outside of the scope of this project.

The Bofetta-Osboren transfer method assumes a piece-wise constant signal and solves for this signal in closed form [35]. In a different approach,the Ablowitz-Ladik discretization method approximates the NLSE with a discrete integrable system [36, 37]. This method is known to produce spurious solitonic components that need to be filtered [34]. The Fourier collocation method expands the problem with a Fourier series and then solves the eigenvalue problem in the Fourier space [34]. This method is only useful for locating the discrete spectral components, but cannot be used to calculate the continuous spectrum. Also, its complexity is quite high compared to other methods. In [38], a new algorithm outperforming the others in speed an accuracy has been proposed. It is based on the Toeplitz matrix transformation. In [39], a fast nonlinear Fourier transform is presented, providing significant lower complexity than other algorithms.

The NFT as described in [34] requires that at the boundary, the signal vanishes, i.e.,

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$\lim_{t \to \pm\infty} q(t) = 0$. Hence, gaps between pulses are required resulting in power drops. In case the signal is periodic, the periodic nonlinear Fourier transform (PNFT) (and its equivalent inverse, the inverse periodic nonlinear Fourier transform (IPNFT)) can be applied. It potentially offers the same possibilities as the NFT for a different type of signals. It has been presented in [40] where it is analyzed analytically, numerical methods are presented and application to a communication system is shown.

2.2 Data Transmission

Numerous transmission schemes have been proposed in previous work. On a broad scale they can be grouped into utilizing the discrete spectrum in contrast to using the continuous spectrum.

2.2.1 Discrete Spectrum

To embed information in the discrete spectrum, one maps data to a predefined set of eigenvalues (spectral components). In [41], information is embedded into the the imaginary part of the eigenvalue. Also, an approximated closed-form posterior probability density function (PDF) is derived, resulting in a discrete-time channel model. In [42], the feasibility of the use of multi-solitons is investigated. Similarly, in [43], a constellation consisting of multi-solitons is used to transmit data over up to 1800 km. Instead of embedding information in the presence or absence of certain eigenvalues, it is possible to modulate the spectral amplitudes of the eigenvalue. In [44], two QPSK constellations are modulated onto the spectral amplitudes of a 2-soliton with only imaginary eigenvalues and transmission over 640 km is demonstrated experimentally. A similar approach is chosen in [45] where the spectral phase of a 2-soliton with symmetric real part is modulated. In [46], the modulation of the phase of seven eigenvalues is demonstrated experimentally. For the existence of a soliton, no attenuation and no noise is assumed. In today's practical systems, these assumptions do not hold. Each component introduces noise and the fiber is lossy which is compensated in regular intervals by erbium-doped fiber amplifiers (EDFAs). The effect of such a non-ideal system is investigated in [47] where it is shown that EDFAs cause the eigenvalues to fluctuate.

2.2.2 Continuous Spectrum

To transmit data in the continuous nonlinear spectrum, the linear spectrum is mapped to the continuous nonlinear spectrum. Hence, modulation formats for the linear spectrum can be directly applied. This concept referred to as nonlinear inverse synthesis (NIS) has been proposed in [48]. and further extended in [49]. In [50] and [51, 52] it is applied to systems with lumped amplification and Raman amplification respectively.

2.2.3 Joint Spectrum

A natural extension is to utilize both discrete and continuous spectrum. It has been proposed in [53] and demonstrated experimentally in [54].

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2.2.4 Constellation Shaping

Constellation shaping is the concept of optimizing the channel input to achieve additional gains. This can be done by optimizing the signal constellation or by optimizing the distribution of the input data and is referred to as geometric and probabilistic shaping respectively. Its biggest advantage is that it only requires additional signal processing and no change of hardware. For this reason, recently, shaping has gotten a large interest for application in optical communications. An overview on different practical implementations of shaping is given in [55]. Also, for an ASK-system, a rate-matched coded modulation scheme operating within 1.1 dB from capacity for the AWGN channel is demonstrated. As an FEC code, the DVBS2 low-density parity-check (LDCP) code, a standard LDCP code with large code length, is used. Regarding the fiber optical channel, the potential of shaping is investigated in [56]. In [57], a 64-QAM constellation is shaped probabilisticly. Experimental verification shows a range increase of about 40 % compared to unshaped 64-QAM. In [58], transmission with shaped constellations is demonstrated experimentally on a transmission between eight German cities.

2.3 Research Progress and Outlook

In previous work, e.g., [47], the bit error rate (BER) has already been investigated resulting in BERs of order 10^{-2} to 10^{-4}. Today's requirement for the BER of an operative system however is 10^{-15}. For NFT-based systems to be of practical use this gap has to be closed. A straightforward way of closing this gap is the use of forward error correction (FEC). For any coding scheme, knowledge of the posterior PDF is essential. However, for most systems with soliton transmission it is unknown. For the special case of amplitude-modulated first-order solitons, an approximated analytical posterior PDF has been presented in [41]. We already showed that we can use this setup to design a bit-interleaved coded modulation (BICM) system with the digital video broadcasting - satellite 2 (DVB-S2) LDCP code and get similar performance results as for the additive white Gaussian noise (AWGN) channel. In fact, we showed that the channel in [41] converges to an AWGN channel. We submitted these results to this year's ECOC conference. Currently, our contribution is under review [59].

For future research in WP2 ESR4, we identify a number of open questions to investigate. Recently, constellation shaping has gotten great attention in the optic community. Since it can be regarded as an add-on to an existing system, we plan to extend our work in [59] with it and investigate whether similar performance improvements as seen in previous work can be achieved in an NFT system. So far, we only considered the imaginary part of the discrete spectrum. We propose to extend this system and, in a first step, use the real part, and later on also the spectral amplitude and phase. However, so far, no posterior PDFs are available for these systems, which are necessary for shaping, coding and coded modulation. Hence, we plan to derive models, either by approximated closed-form solutions or by using a Gaussian mixture model to approximate results from simulation and experiments. Once we established a posterior PDF, we propose to investigate the performance and find possible improvements.

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